THE 2012 HANS CLOOS LECTURE. PHYSICO-CHEMICAL THEORY OF EFFECTIVE STRESS IN SOILS

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Abstract: The paper presents a new theory of effective stresses in soils. Unlike the well-known theory of Karl Terzaghi, it considers soil as a structured system with operating external and internal stresses of gravitational and physico-chemical origin. Stresses calculated with consideration of both external and internal forces correspond to the actual effective stresses existing in a structured porous body. The paper characterizes both groups of stresses and provides the equations for calculating total actual effective stresses in soils.

Key words: *effective stress; pore pressure; contact interaction; wedging forces of hydrate films; capillary pressure; open and closed porous systems*.

INTRODUCTION

The theory developed by Terzaghi in the early 1920s forms one of the fundamentals of modern soil and rock mechanics (Terzaghi 1925). According to this theory, all changes in soils are related to stress transmitted to the soil skeleton, which may be represented as: $\sigma' = \sigma - u$, where σ' is the effective stress, σ is the total stress and u is the neutral stress transmitted to the pore water. Terzaghi's theory is successfully used for solving different problems in soil mechanics and geotechnics. At the same time, practical experience shows that it cannot be adequately applied to fine-grained, clay soils. The explanation is that the theory in question considers a soil as a homogenous unstructured body and does not take into account the following factors:

- stress distribution at the contacts between structural elements;

- evolution of physico-chemical processes operating in the soil and inducing the development of internal stress;

- the presence of thin bound-water films at the contacts;

- the dependence of the neutral (porous) pressure on the pore openings in the soil.

1

The attempts to modify Terzaghi's theory were made by Skempton (1960), Bishop (1960), de Boer (2000), Mitchell and Soga (2005) and many others. Despite intense research in this field, the problem still remains unsolved.

The Russian scientists P.A. Rebinder and B.V. Deryagin (members of the Russian Academy of Sciences) followed by their numerous colleagues contributed substantially to the study of the behaviour of structured heterogeneous bodies. Their work promoted the development of a new scientific area, i.e., physico-chemical mechanics of fine-grained systems, which includes contact interactions and the theory of wedging effects of boundary hydrate films. Achievements in these scientific schools permit interpretation of the theory of effective stresses in soils in terms of physico-chemical mechanics.

Stresses arising in soils may be of two genetic types: external and internal stresses. External stresses are produced by gravitational forces. They include geodynamic (σ_g), hydrostatic (σ_h), hydrodynamic (σ_{hd}), and technogenic (σ_t) stresses caused by engineering structures. The former three types are natural stresses produced by the overlying soil mass, the weight of the water column filling the soil pores and the water moving in pores transmitting stress from the viscous friction of water at the boundary with the surface of mineral grains to the soil solid skeleton. The latter type of stress arises from the impact of engineering structures and other technogenic loads on soil.

The internal stresses, which are of physico-chemical origin, arise inside the soil at the contacts between structural units. They include stresses caused by capillary forces (σ_c) and the wedging impact of thin hydrate films ($\Pi_{(h)}$).

The Terzaghi theory takes into account only external forces and effective stresses produced by them (σ'). The consideration of the total effect of both external and internal forces permits us to determine actual effective stresses operating in a structured heterogeneous body (σ''). *Under the actual total effective stress of a structured body (σ'') it should be taken to mean the averaged effective stress value caused by external and internal forces.*

$$\sigma'' = \sigma_a + \sigma_h + \sigma_t - u - \Pi_{(h)} + \sigma_c \qquad\qquad (1)$$

where $\sigma_a + \sigma_h + \sigma_t - u$ are external stresses and

$-\Pi_{(h)} + \sigma_c$ are internal stresses.

The procedure of assessing external stresses is well developed in rock and soil mechanics; it is described in a number of monographs and guidebooks (Dashko, 1987; Mironenko & Shestakov,

2

1974; Ukhov *et al.*, 1994). Internal stresses in soils are discussed in publications to a lesser extent. Therefore, the forces causing the internal stresses in soils are considered below.

INTERNAL STRESS

Internal stresses operate in a fine-grained soil system. They are induced by various surface phenomena appearing at the mineral - air and mineral - liquid boundary. These phenomena embrace the formation of a double electric layer and an electrostatic field of particles, the local osmotic processes, the formation of hydrate films of bound water on the particle surface, the formation of capillary menisci, recharging of clay particle split edges upon different pH, etc. Stresses induced by the above-mentioned processes are scrutinized below.

Capillary pressure

The wetting phenomenon of atomic-molecular origin is observed in liquid-solid interaction. It implies that an angle θ is formed between the liquid surface and the solid body, which is named *the wetting angle.*

Wetting is the reason for the formation of capillary menisci in pores and capillary moisture condensation. *Capillary condensation* occurs on warping of the liquid surface and formation of a meniscus at the contact with gas or other liquid. Condensation results from the fact that the relative vapour tension above the concave meniscus is lower than that above the flat surface.

The pressure difference over the warped (P_0) and flat (P_s) surfaces produces capillary pressure (Pc) described by the Laplace formula:

$$P_c = P_o - P_s = \frac{2\sigma_w}{r} = \frac{2\sigma_w \cos\theta}{R_o} \, , \tag{2}$$

where r is the curvature radius of the meniscus, σ_w is the surface tension of water, θ is the wetting angle and R_0 is the capillary radius.

The capillary pressure is positive in the case of a concave meniscus and is negative in case of a convex meniscus.

Capillary menisci and capillary pressure are formed upon vapour condensation in soil pores upon their relative saturation $P/P_s \leq 1$. This is often observed for soils in the aeration zone. The formed menisci contact particles and increase the structural strength of soil.

A meniscus of radius r_o at the contact of two particles represents a rotation surface characterized at each point by the curvature radii r_1 and r_2, with $1/r_1 + 1/r_2 = $ constant (Fig. 1).

3

The developing meniscus ties soil surfaces thus causing the manifestation of capillary contracting forces between particles:

$$F_c = \pi r_1 \sigma_w \left(1 - \frac{r_1}{r_2} \right) . \qquad (3)$$

The contracting force F_c depends on the amount of water in a pore. With a decreasing amount of liquid in the pores on clay drying, the capillary contracting force increases (Fig. 1a) to reach the maximum upon $r_1 \rightarrow 0$. The capillary contracting force decreases with the growing amount of liquid (Fig. 1b) to disappear completely on the filling of a pore with water.Further moistening results in the filling of capillary pores with water and the capillary rise of water along the capillary pore network arranged in vertical channels (Fig. 2). In doing so, the menisci of pore angles disappear and are replaced by the menisci of capillary pores, which control the lifting force and the height of water capillary rise. The lifting force of menisci is transmitted to the soil skeleton and increases the effective stress.

The height of capillary rise (h_c) may be calculated from the Laplace formula:

$$h_c = \frac{2\sigma_w Cos\theta}{R_0 \rho_w g}, \qquad (4)$$

where R_0 is the capillary radius, r is the meniscus curvature radius, σ_w and ρ_w are the surface tension and liquid density, respectively and g is the gravity acceleration. For calculations, the R_0 value is taken to be equal to the average pore dimension.

Since the entire raised water mass (m_w) transmits its weight to the pore walls, it determines the total tension produced by capillary forces in the soil skeleton:

$$\sigma_c = m_w h_c g . \qquad (5)$$

Thus, the value and manifestation specifics of capillary forces depend on the amount of water in the soil. The contracting force of pore angle menisci is formed on the contacts between particles upon wetting close to the maximal hygroscopic moisture (W_{mg}). Further wetting of the soil results in the filling of capillary pores with osmotic and capillary water and induces the capillary water to rise along tubular capillary pores. With the continuing capillary rise, the contracting stress caused by the weight of capillary raised water increases and replaces the forces of pore angle menisci. This effect increases up to the moisture content of capillary water capacity (W_c), after which the capillary forces start mitigating to disappear completely upon the total water saturation of the soil (the moisture content of total water capacity - W_{sat}).

4

Wedging pressure of hydrate films: components of wedging pressure

The excessive pressure, which is manifested in the interphase liquid layer in the contact spacing upon its thinning or thickening is named *wedging pressure* and is designated as Π(h).

The idea of wedging pressure in thin boundary films was first introduced by Deryagin & Kusakov (1936). Later, a great contribution to the investigation of boundary films was made by Deryagin & Churaev (1972), Deryagin & Zorin (1955), Deryagin & Titievskaya (1953), as well as by the work of Israelashvili (1978), Israelashvili & Pashley (1983), Marcelja & Radic (1976), Pashley & Kitchener (1979).

The wedging pressure may be either positive, i.e., preventing the film thinning, or negative, i.e., favouring the film thinning. The value of the wedging pressure, Π(h), and the thickness of the boundary films, h, are controlled by the input of surface attractive and repulsing forces of various origins. To a first approximation, this input may be considered as an additive one, consisting of the following components (Deryagin & Churaev, 1984):

$$\Pi(h) = \Pi_m(h) + \Pi_e(h) + \Pi_s(h), \tag{7}$$

where Π_m is the molecular component produced by the dispersion interaction of the solid surface through a thin liquid film, Π_e is the electrostatic component resulting from overlapping of diffusive ionic layers of charged particle surfaces and their repulsion, and Π_s is the structural component of wedging pressure caused by the overlapping of adsorption liquid layers with the modified structure.

For the first time, the general theory of molecular interactions was formulated by London (1937). Later, Hamaker, Verwey and Overbeek contributed significantly to its development, having calculated the molecular attraction between fine-grained particles.

For two particles divided by a thin flat spacing *(h)* filled with water, *the molecular component of wedging pressure (Π$_{mol}$)* can be found from the following equation:

$$\Pi_{mol} = -\frac{A^*}{6\pi h^3}, \tag{8}$$

Where A* is the complex constant of Hamaker.

For symmetrical films, the molecular component of wedging pressure is negative. The molecular forces tend to make the hydrate film dividing solid surfaces thinner.If two fine-grained particles surrounded by a double electric layer approach one another, their ionic atmospheres become overlapped. This effect produces *the electrostatic component of wedging pressure.*

5

The theory of electrostatic interaction of diffusive ionic layers surrounding fine-grained particles was developed by Deryagin, Landau, Verwey, Overbeek and many others in the 1930s-1940s. At present, it is the basis for the quantitative theory of stability of highly-dispersed systems (DLVO theory).

According to this theory, the energy of electrostatic wedging increases with thinning of the dividing spacing between particles. The repulsing energy of two plane-parallel particles with low surface potential (φ_0) is found from the following equation (Shchukin, Pertsev, Amelina, 2007):

$$\Pi_e \approx \frac{4z^2 e^2 \varphi_0 n_0}{kT} e^{-\text{æh}}, \tag{9}$$

where z is the charge of counterions, e is the electron charge, kT is the energy of heat movement of ions, æ is the parameter characterizing the ionic atmosphere thickness $\delta = 1/\text{æ}$, h is the thickness of the spacing between particles and n_0 is the concentration of ions in the spacing.

For a highly charged surface, the electrostatic component of wedging pressure does not depend on φ_0 value and can be found from the following equation (Shchukin, Pertsev, Amelina, 2007):

$$\Pi_e \approx 64 n_0 kT e^{-\text{æh}}. \tag{10}$$

Unlike the molecular component, electrostatic forces are positive, which corresponds to particle repulsion.

On the moving together of clay particles and overlapping of adsorbed water films, the repulsion forces caused by the structural-mechanical properties of adsorbed films appear at the contact of the particles (Fig. 3). The destruction of the adsorbed water film in the overlap zone is accompanied by changing free energy and developing wedging pressure. The developing forces that prevent approach of particles were named as the *structural component of wedging pressure* (Deryagin & Churaev, 1984; Deryagin, 1986).

The existence of structural wedging pressure and its dependence on the thickness of adsorbed film were proved experimentally by Deryagin & Zorin (1955) and Deryagin & Churaev (1972) by the example of the watered surface of quartz and glass. Later, similar data were obtained for mica plates (Izraelachvili & Adams, 1978) and montmorillonite (Viani *et al.*, 1983). The Australian physico-chemists Izraelachvili & Pashley (1983) related the structural wedging effect to water molecules, which hydrate cations, and designated it as the hydration effect.

Marcelya & Radic (1976) were the first to calculate the wedging pressure of adsorption films $\Pi_s(h)$ on the basis of the empirical relationship:

$$\Pi_s(h) = K/sh(h / (2l)), \qquad (11)$$

where l and K are empirical parameters.

For $h > l$, there is exponential dependence of the structural repulsive forces on the spacing thickness:

$$\Pi_s(h) = 4K\exp(-h/l). \qquad (12)$$

K and l values were tabulated by Deryagin & Churaev (1984) on the basis of analysis of experimental data obtained by various authors for glass, quartz and mica surfaces.

Isotherms of wedging pressure of hydrate films

The molecular, electrostatic, and structural components of wedging pressure operate simultaneously in the hydrate film dividing particles and obey different rules depending on film thickness.

The wedging pressure equation, $\Pi(h)$, for particles with a low surface potential (φ_0) is (Shchukin $et\ al.$, 1982):

$$\Pi(h) = \frac{4z^2e^2\varphi_0^2 n_0}{kT}e^{-\text{æh}} - \frac{A^*}{6\pi h^3}, \qquad (13)$$

and for particles with high surface potential:

$$\Pi(h) = 64n_0 kTe^{-\text{æh}} - \frac{A^*}{6\pi h^3}, \qquad (14)$$

where z is the charge of counter-ions, e is the electron charge, kT is the energy of thermal motion of ions, æ is the parameter characterizing the thickness of the DEL (double electric layer) ionic atmosphere ($\delta = 1/\text{æ}$), h is the spacing size between particles, n_0 is the concentration of ions in the spacing and A* is a complex Hammaker constant.

Distinguishing wedging pressure produced by the structural component $\Pi_s(h)$ in the general wedging effect faces certain difficulties due to an inadequate level of theoretical knowledge in

this area. Hence, experimental data are used for quantitative estimates of the structural component.

The dependence between the total action of wedging forces in the boundary hydrate film in the gap between solids and the film thickness h is named *a wedging pressure isotherm* (Deryagin & Churaev, 1984). Fig. 4 shows the qualitative pattern of this relationship.

Those isotherm sections, where $\partial\Pi/\partial h < 0$ condition is met, correspond to the stable state of films. For example, for the isotherm plotted in Fig. 4, stable films have thicknesses ranging from h_2 to h_0 and $< h_1$. In this regard, the film stability intervals h_0-h_2 and $h < h_1$ are named *β and α branches of the wedging pressure isotherm* (Deryagin & Churaev, 1984). The former branch fits thick films with h reaching hundreds nm; whereas the latter corresponds to thin films with their thickness rarely exceeding 10 nm. The isotherm intervals from h_1 to h_0 and $h > h_2$ correspond to the unstable state of films, as $\partial\Pi/\partial h > 0$ there. The film thickness fitting the α and β branches of adsorption isotherms characterize thermodynamic equilibrium, where interacting particles can fix themselves due to surface forces to form volume frameworks.

Experimental studies of wedging pressure of hydrate films

Theoretically, the wedging pressure isotherm $\Pi(h)$ may be calculated only for some cases and with limited accuracy. To obtain more reliable data, the experimental methods are applied for determining the wedging pressure of α- and β-films.

The first experiments on finding wedging pressure were preformed by Deryagin & Kusakov (1936). They measured pressure in a thin hydrate film at the contact between a solid surface and an air bubble. The conditions were created under which floating air bubbles of various radii R_0 approached a smooth horizontal surface (Fig. 5). The floating air bubble formed a contact area with the solid surface of r_1 radius and a boundary film of h thickness. The wedging pressure in the film was estimated from the capillary pressure of the bubble by measuring the bubble radius and the boundary film thickness: $\Pi = 2\sigma_w/R_0 = \Delta P_\kappa$, where σ_w is the surface water tension, ΔP_κ is the capillary pressure of bubble and R_0 is the bubble radius. The experiments performed allowed assessment of the wedging pressure in the area of positive values $\Pi(h)$ approximating to $\sim10^3 \div 10^4$ dyne/cm^2.Deryagin & Churaev (1974) generalized the experimental data on the study of the wedging effect produced by β-films obtained by various researchers. Fig. 6 shows the results of this analysis as applied to wetting water films and water solutions of symmetrical electrolytes formed at silicate surfaces.

As seen from the data obtained, the wedging effect of a water film for silicate surfaces varies, depending on its thickness, from $0.1 \cdot 10^4$ to $1.4 \cdot 10^4$ dyne/cm^2. According to Deryagin & Churaev (1974), under favourable physico-chemical conditions, the maximal wedging pressure of β-film (upon its critical thickness) may reach $3 \cdot 10^6$ dyne/cm^2.

STRESS ON CONTACTS BETWEEN PARTICLES

For assessing the total actual effective stress in soils it is necessary to estimate the effective stress at the contacts. For this purpose, it is necessary to know: (1) the type of contact; (2) the number of contacts per square unit of any area inside the soil; (3) the area of contacts; (4) the total effective stress on the unit contact produced by external and internal forces.

Types of contacts

Distinguishing contacts by their energetic state appears to be of particular interest. Despite a great diversity of geometrical types, origin, size and shapes of structural elements forming the porous bodies, three energy types of contacts may be distinguished that are met in disperse coherent systems: coagulational, transitional (point) and phase contacts (Fig. 7) (Osipov, 1979; Osipov & Sokolov 1974; Rebinder, 1958, 1966; Sheludko, 1958; Shchukin *et al.* 1982). Each type of contact is characterized by a certain mechanism of formation, the origin of forces operating in the contact zone and the specific features of effective stress transmission.

Coagulational contacts are those formed between the interacting particles with the preserved thermodynamically stable interlayer of bound water at the contact (Rebinder, 1958, 1966; Yaminskii*et al.*, 1982) (Fig. 7a). In nature, coagulational contacts are met in water-saturated, non-compacted or weakly compacted clay deposits, such as mud, weakly lithified silt and swelling clay of plastic or fluid consistency.

Surface hydrate films preventing the approaching together of particles play the most important role in the development of coagulational contacts. A thermodynamically stable state at coagulational contacts is achieved by the thickness of hydrate films fitting α and β branches in the adsorption isotherm. Accordingly, coagulational contacts are usually subdivided into *far-field coagulational* (corresponding to the β-film thickness) and *near-field coagulational* contacts (corresponding to the α-film thickness).

Transitional contacts are formed in the course of clay compaction in a water-saturated state or upon their dewatering on drying. Punching (destruction) of boundary hydrate films takes place in

9

some rugged (bulged) spots of contacting surfaces under the impact of external and internal factors. Within a small square of direct contact between particles (Fig. 7b), new point contacts (in terms suggested by Rebinder) are formed.

The behaviour of these contacts is determined by both the wedging effect of hydrate films (in the coagulational contact zones) and the stronger chemical and ion-electrostatic forces (in the zones of direct contact of particles); therefore, this form of contact is called a *transitional contact* (Osipov, 1979; Osipov & Sokolov, 1974, 1985; Sokolov, 1973).

The strongest *phase contacts* are formed in the course of cementation or with the rising geostatic pressure and temperature (Fig. 7c). This type of contact is based on the ion-electrostatic and chemical forces, under the impact of which clay particles are transformed into coarser clay ultra- and microaggregates - crystallites. By their origin, the formed contacts are similar to the grain boundaries in a polycrystalline body.

Phase contacts show high strength sometimes exceeding the strength of contacting minerals proper. They are widespread in cemented clay deposits such as argillites, siltstones and schists, which are typical solids manifesting the elastic deformation upon the application of an external load and the fragile destruction on achieving the limit strength. Unlike transitional contacts, phase contacts are almost non-hydrateable, which provides for water resistance and the absence of swelling capacity in these rocks.

Number of contacts

For assessing the number of contacts, any horizontal surface of (s) square inside a porous body is assumed, which is perpendicular to the applied external stress. It is assumed that all individual contact areas between structural elements are also normally directed to the operating stress. The problem is to find the number of contacts per unit square of horizontal surface.

At present, several calculation schemes, that is, models of a dispersed porous body are developed to estimate the number of contacts per unit square of horizontal surface. The amount of contacts in these models is controlled by the size and shape of particles and the mode of their package, which is closely related to porosity.

The simplest model is *the globular model* suggested by Rebinder *et al.* (1964) for the structures composed of spherical particles. To build a globular model, the straight chains consisting of balls of equal diameter touching each other are used (Fig. 8a). Chains stretch in three dimensions and

10

cross to form structural nodes. The package mode is described by a structural parameter N, which stands for an average number of particles between two nodes. In such a model, porosity n is related unambiguously to parameter N. This relationship is expressed as (Amelina & Shchukin, 1970):

$$\frac{1}{N^2} = f(n) \tag{15}$$

The number of contacts per unit square of contacting surface (χ) is related to parameter, N, and an average radius of structural element, r, according to the following relationship (Amelina & Shchukin, 1970; Rebinder *et al.* 1964):

$$\chi = \frac{1}{4r^2N^2} \tag{16}$$

The main drawback of the globular model is the fact that it does not take into account size diversity and shape anisotropy of structural elements composing clay. Therefore, for polydisperse systems, Sokolov (1991) proposed a *bidisperse globular model*, which permits assessment of the number of contacts in the system composed of coarse (R radius) and fine (r radius) particles (Fig. 8b). According to this model, a total number of contacts is equal to the number of contacts between coarse particles (χ_R), multiplied by the number of contacts between fine particles (χ_r), occurring within a contact area between coarse particles. The χ_R and χ_r values are found from the following formulae, respectively:

$$\chi_R = \frac{3z(1-n)}{8\pi R^2} \tag{17}$$

$$\chi_r = \frac{\rho_R \varphi_r R^2}{2\rho_r \varphi_R r^2}, \tag{18}$$

where $\rho_R, \rho_r, \varphi_R, \varphi_r, R\ u\ r$ are density, content, and average equivalent radius of coarse and fine particles, respectively; z is the coordination number, and n is porosity.

A total number of contacts in the bidisperse system is found as follows:

$$\chi = \chi_R \chi_r = \frac{3z(1-n)\rho_R \varphi_r}{16\pi r^2 \rho_r \varphi_R} \tag{19}$$

To describe clay composed of flat anisometric particles, the model of *twisting playing card house* was proposed (Sokolov, 1985, 1991).

For estimating the compaction degree of soils, the suggested model (Fig. 9) uses the parameter θ, which stands for the average angle between particles. The porosity is related to the θ value via the following equation:

11

$$n = 1 - \frac{K \cdot b/a}{Sin\ \theta + (K \cdot b/a)} , \qquad (20)$$

where K is a coefficient (for discs, $K = 3\ \pi/4$), a and b are the average diameters and thicknesses of discs, respectively.

The considered model permits assessment of the number of contacts per unit square of contacting destruction area (χ^θ) depending on the average angle between the clay particles (θ):

$$\chi^\theta = \chi^{90}/Sin\ \theta . \qquad (21)$$

The number of contacts χ^{90}, when the cell-forming particles are perpendicular to each other ($\theta = 90°$), is determined as $\chi^{90} = 2/a^2$, where a is the particle length (diameter).

With the consideration of the above discussion, equation (21) may be written as:

$$\chi^\theta = 2/Sin\theta \cdot a^2 . \qquad (22)$$

Parameters a and θ can be estimated on the basis of quantitative analysis of micro-structural clay images obtained from the scanning electron microscope.

Area of contacts

Coagulational contacts. The area of coagulational contact between two spherical particles ($a_{c(c)}$) depends on their radii (R) and the hydrate film thickness (h) (Fig.10a). Parameter (a_c) characterizes the overlapping zone produced by hydrate films of contacting particles within which the wedging pressure exists. For two spherical particles, the unit contact area is found from the equation:

$$a_{c(c)} = 2\pi\ R(h - \frac{l}{2}) , \qquad (23)$$

where h is the hydrate film thickness on the particle surface outside the contact, l is the hydrate interlayer thickness on the contact with overlapping films of interacting particles and R is the particle radius.

The contact area between two flat particles of the same size situated at an angle θ (Fig.10a) is found in a different way:

$$a_{c(c)} = \left[\min\left(\frac{h}{Sin\ \frac{\theta}{2}} - \frac{l}{Sin\ \theta}, a \right) + \min\left(\frac{h}{Sin\ \frac{90^0 - \theta}{2}} - \frac{l}{Sin\ (90^0 - \theta)}, H + 2h \right) \right] \cdot b . \qquad (24)$$

where a and b are the width and length of two interacting flat particles.

12

Expressions $\min\left(\dfrac{h}{\sin\frac{\theta}{2}} - \dfrac{l}{\sin\theta}, a\right)$ and $\min\left(\dfrac{h}{\sin\frac{90^0-\theta}{2}} - \dfrac{l}{\sin(90^0-\theta)}, H = 2h\right)$

mean that the lowest of considered values are taken, that is, the following condition is met:

$\min(a, b) = a$, if $a \le b$, or $\min(a, b) = b$, if $a \ge b$.

For $\theta \to 0$, that is, particles are parallel and overlap each other completely, the contact area will evidently be controlled by a and b dimensions of overlapping particles.

The area of coagulational contact of a spherical particle with a flat one can be found from the equation:

$$a_{c(c)} \approx 2\pi R(h\text{-}l) \tag{25}$$

Transitional contacts. A total area of transitional contact a_c is composed of a total area of immediate tangency ($a_{c(s)}$) sites and the coagulational sites ($a_{c(c)}$) (Fig. 10b). Assessing the area of the latter ('dry') sites (devoid of hydrate film) appears to be most difficult.

To estimate the maximal area of 'dry' sites within a transitional contact, the tensile strength of these contacts and their hydration capacity may be used. For this purpose, the most important feature of a transitional contact can be used, that is, its metastability or ability to pass into a coagulational contact under certain conditions.

The experiments prove (Osipov & Sokolov, 1974) that the transitional contact between two particles of a micron size loses its reversibility on reaching a strength of $\sim 3\times10^{-7}$N. At a higher strength, transitional contacts behave as irreversible phase contacts. The value 3×10^{-7} N may be evidently taken as an upper boundary of transitional contact strength. As was already noted, transitional contacts are formed via chemical (valence) bonds. Assuming the unit valence bond equal to $e^2/4\pi\varepsilon_0 b^2 \approx 10^{-9}$ N, (where e is the electron charge, ε_0 is the dielectric constant equal to 8.85×10^{-12} C^2/N·m^2, b is the distance between atoms in a crystalline cell ~ 0.1 nm), it is obtained that for the formation of a transitional contact with the maximal strength 3×10^{-7} N, 3×10^2 bonds are needed. With the amount of chemical bonds $<3\times10^2$ the formed transitional contact manifests metastability. Taking into account the crystalline lattice of basal surfaces in phyllosilicates, it can be calculated that 3×10^2 bonds can be formed within a square of 35 nm^2 in size.

Knowing the $a_{c(s)}$ value, and having calculated a total contact area (a_c) from one of the equations (23-24), it is possible to determine a share of 'dry' and coagulational areas in the total contact area as $a_{c(s)} / a_c$ for the former, and ($1 - a_{c(s)} / a_c$) for the latter.

Phase contacts. The area of phase contacts may vary widely depending on crystallization development in the course of rock lithification with the formation of large clay crystallites and

13

the degree of clay cementation. The minimal area of phase contact, which becomes irreversible on unloading and wetting of rock, is no less than 35 nm^2, whereas the maximal area fits the area of basal planes in clay minerals (Fig. 10c).

Effective contact stresses

Water-saturated soils. At the coagulational contacts (Fig. 10a) thin hydrate interphase films produce the wedging pressure, which is characterized by a $\Pi(h)$ value. Multiplying the wedging pressure by the contact area ($a_{c(c)}$), a total value of inner physico-chemical forces acting on a coagulational contact may be assessed:

$$\sigma_c = \Pi(h) \cdot a_{c(c)}. \tag{26}$$

Apart from the inner forces, the effective stresses produced by general external impacts equal to σ'/χ, where χ- is the number of contacts per unit area of contact surface, are transmitted to each contact.

Then the effective stress value at the unit coagulational contact will be determined as a difference between the external and internal forces:

$$\acute{\sigma}_c = \acute{\sigma}/\chi - \Pi(h)\, a_{c(c)} \tag{27}$$

At the transitional contacts the interphase hydrate layer is absent at the sites of immediate touch of particles ('dry sites'), therefore, effective stress at these sites is transmitted directly to the soil skeleton. On coagulational sites, the wedging pressure of hydrate film is present, which reduces the external effective stress at these sites. The total effective stress on transitional contacts is summed up from the stresses transmitted to 'dry' and coagulational sites of the contact (Fig. 10b).

Effective stresses transmitted to the sites with direct contact of particles ($\acute{\sigma}_{c(s)}$), may be calculated from the following equation:

$$\acute{\sigma}_{c(s)} = \frac{\sigma' a_{c(s)}}{\chi a_c}, \tag{28}$$

where $\acute{\sigma}$ is a total external effective stress, $a_{c(s)}$ is a total area of 'dry' sites at the transitional contact, a_c is a total area of transitional contact and χ is the number of contacts per unit area of horizontal surface.

Similarly, the effective stresses may be estimated at the coagulational sites of the contact ($\acute{\sigma}_{c(c)}$):

14

$$\acute{\sigma}_{c(c)} = \left(\frac{\sigma'}{\chi} - \Pi(h)a_{c(c)} \right) \cdot \left(1 - \frac{a_{c(s)}}{a_c} \right), \tag{29}$$

where $\Pi(h)$ is the wedging pressure of the hydrate film.

Then, the total effective stress at the transitional contact will be equal to:

$$\acute{\sigma}_c = \acute{\sigma}_{c(s)} + \acute{\sigma}_{c(c)} = \frac{\sigma'}{\chi} - \Pi(h) \ a_{c(c)} \left(1 - \frac{a_{c(s)}}{a_c} \right) \tag{30}$$

At the phase contacts, the contacts receive the entire effective stresses and transmit them to the soil skeleton (Fig. 10c). The influence of physico-chemical factors on the distribution of effective stresses is negligible there. Hence, the Terzaghi law is successfully applied for coarse fragmentary and the bulk of sandy soils, as well as for fractured rocks including cemented clay deposits such as argillites, siltstones and schists.

Unsaturated soils. Clay soils occurring in the aeration zone above the capillary fringe are usually characterized by transitional contacts. Taking this fact into consideration and using equation (3), the value of effective stresses at an individual contact, takingaccount of the capillary menisci effect, may be obtained:

$$\sigma_c' = \frac{\sigma'}{\chi} - \Pi(h) \ a_{c(c)} \left(1 - \frac{a_{c(s)}}{a_c} \right) + \pi r_1 \sigma_w \left(1 - \frac{r_1}{r_2} \right), \tag{31}$$

where r_1 and r_2 are the menisci radii in the contact spacing and σ_w is the surface tension of water.

The effective contact stresses operating in clay below the capillary fringe are calculated taking into account the water mass pressure (m_w) capillary raised to the height h_c on the soil skeleton. Then, for deposits with prevailing coagulational contacts, the value of contact effective stresses taking into account the capillary forces will be:

$$\sigma_c' = \frac{\sigma'}{\chi} - \Pi(h)a_{c(c)} + \frac{h_c m_w g}{\chi}, \tag{32}$$

where g is the gravity acceleration.

Where the transitional contacts prevail in the deposit, the equation is modified as:

$$\sigma_c' = \frac{\sigma'}{\chi} - \Pi(h) \ a_{c(c)} \left(1 - \frac{a_{c(s)}}{a_c} \right) + \frac{h_c m_w g}{\chi} \tag{33}$$

15

Pore pressure

In equations (27-33), the value of total external effective stress $\sigma' = \sigma - u$ is used. According to the Terzaghi theory, the total stresses (σ) appearing in the soil are transmitted not only to the mineral skeleton to produce the effective stress there (σ'), but also to the pore water (u). This induces the manifestation of *pore pressure* upon total soil saturation with water. Development of pore pressure and its alteration under the impact of external stresses depends on the pattern of pore space and the pore interrelations. Therefore, clay systems with open and closed pore space are considered next.

Open pore system. Sandy loam, silty loessic deposits and light loam with a low and medium degree of lithification are usually characterized by open porosity. In these systems pores are interconnected and pass water, with pore water bearing a part of the external stress (Fig. 11a). On applying the external load, the pore pressure (u) appears in the soil, which dissipates gradually in the course of consolidation to become equal to the external hydrostatic pressure. Therefore, at the stage of filtrational consolidation, the external effective stresses in these soils are described according to the Terzaghi theory. Upon completion of consolidation and achieving thermodynamic equilibrium, the pore pressure (u) dissipates or becomes equal to the external hydrostatic pressure (u_o) and the effective stress acquires a value equal to the total external stress $\sigma' = \sigma$. From the aforesaid discussion it may be assumed that for the calculation of actual effective stresses in unconsolidated soils with the open porosity, the expression $\sigma - u$ should be used instead of σ' in formulae; whereas σ should be used for consolidated or overcompacted soils.

Closed pore system. Many clay soils, especially those containing smectite, become a completely closed porous system upon consolidation compaction. This is related to the clay transition to the secondary consolidation stage. The bulk of porous space in such clay consists of ultramicropores and fine micropores, being filled with bound (adsorbed and osmotic) water. Free water is preserved only in large micropores and macropores (Fig. 11b).

The bound water is known to be influenced by the surface forces of particles and to manifest an elevated viscosity. Therefore, the bound water films can be involved in viscous flow only on achieving the initial shear stress (τ_o).

The compressive compaction produces pore pressures in the isolated pores containing free water, which does not dissipate but continues rising with the continuing soil compaction. Free and partially-bound pore water is pressed out to the open pore space only when the water head in the pore space exceeds the pressure of the initial filtration gradient (I_H). Under these conditions,

the shear resistance of external layers of bound water is overcome in sealed fine pores, free water rushes through them, and water pressures in large micropores become equal to I_H.

Thus, the pore pressure in clay with a closed pore system differs from the external hydrostatic pressure. It may decrease stepwise in the course of consolidation, but it practically always remains higher than the pressure in the open pore space. This is the reason for the long secondary consolidation of such clay, which may last sometimes scores and even hundreds of years. At the same time, the pore pressure in clay with a closed porous system does not exceed the pressure of the initial filtration gradient. Therefore, in laboratory tests, $\sigma' = \sigma - u'$, where u' is the critical pressure in pores equal to the initial filtration gradient, can be taken as a value of total effective stress.

ACTUAL TOTAL EFFECTIVE STRESSES IN SOILS

Actual total effective stresses in a soil my be calculated with the consideration of effective contact stresses obtained above, the data on the number of contacts per unit area of soil cross-section, as well as the equation for pore pressure in open and closed porous systems. Depending on the prevailing types of contacts and the degree of soil water-saturation, the calculation equations will differ.

Water-saturated soils. Mud and weakly lithified clay with coagulational contacts and an open pore network will manifest the effective contact stress found from equation (27), and a pore pressure equal to u (at the stage of filtrational consolidation) or $u = 0$ (at the stage of secondary consolidation). In this case, the equation for the actual total effective stress in these soils is:

$$\sigma'' = \chi \left[\frac{\sigma - u}{\chi} - \Pi(h) a_{c(c)} \right] = (\sigma - u) - \chi \Pi(h) a_c \tag{34}$$

where χ is the number of contacts per unit area of a horizontal square, $\Pi(h)$ is the wedging pressure of hydrate films, $a_{c(c)}$ is the coagulation contact area, σ is the total external stress and u is the pore pressure.

Clay soils either containing a lot of clay minerals, of smectitic composition, in particular, or being subject to a certain compaction in the course of lithogenesis with coagulational structure preserved, may have a closed pore space structure. In some isolated pores of this structure, the pore pressure (u') may form which will be permanently kept not lower than the pressure of the initial filtration gradient in this clay (I_H). Hence, the actual total effective stress in this clay (σ'') may be determined from the following expression:

$$\sigma'' = \left[\sigma - u' \right] - \chi \Pi(h) a_{c(c)} \tag{35}$$

17

The stress state of soils with transitional contacts is controlled by the existing sites of direct contact between particles and the coagulational sites. For such a contact, stresses are found from equation (30). The bulk of soils with transitional contacts underwent compaction in the course of lithogenesis. Therefore, they are treated as overconsolidated soils in engineering practice. The pore pressure in overconsolidated soils may develop only when the load exceeds their natural compaction pressure, which is most unlikely in engineering practice, because the natural historical load of compacting water-saturated soils with transitional contacts is equal to 20-30 MPa. Therefore, the general external stress $\acute{\sigma} = \sigma$ should be taken as an effective external stress in these soils. Then, the expression for the actual total effective stress in water-saturated soils with transitional contacts will be:

$$\sigma'' = \sigma - \chi \Pi(h) a_{c(c)} \left(1 - \frac{a_{c(s)}}{a_c} \right)$$

(36)

where $a_{c(s)}$ is the area of immediate ('dry') contacts.

Soils and rocks with phase contacts are the cemented or crystalline formations, where contacts are similar to the boundaries between the grains in a polycrystalline body. With fracturing absent, effective stresses in these rocks fit the general stress $\acute{\sigma} = \sigma$. In water-saturated fractured rocks, effective stresses are calculated taking intoaccount wedging water pressure.

Unsaturated soils. The actual total effective stress in unsaturated clay depends on their moisture content. For the moisture content below W_{mg}, the compressive forces of capillary menisci operate in the soil. Taking into consideration that transitional contacts prevail at such a moisture content, the value σ'' is found as follows:

$$\sigma'' = \sigma - \chi \Pi(h) a_{c(c)} \left(1 - \frac{a_{c(s)}}{a_c} \right) + \chi \pi r_1 \sigma_w \left(1 - \frac{r_1}{r_2} \right), \tag{37}$$

where r_1 and r_2 are the radii of menisci in the contact spacing and σ_w is the surface tension of water.

With an increasing moisture content and capillary saturation of the soil, the compressive effect of the capillary raised water grows. The actual total effective stress for this soils can be found from the following equation:

18

$$\sigma'' = \sigma - \chi\Pi(h)a_{c(c)}\left(1 - \frac{a_{c(s)}}{a_c}\right) + h_c m_w g \ , \tag{38}$$

where m_w is the mass of capillary raised water, g is gravity acceleration, and h_c is the height of capillary water rise.

CONCLUSION

The presented data prove that the stresses operating in soils and rocks appear to be a complex derivative of external impacts and internal processes, stretching far beyond a simple consideration of the applied load and pore pressure as was earlier suggested by Terzaghi. The structural specifics of soils, their composition, the degree of lithification and water saturation is of great importance for the assessment of the stress state. Contacts between structural elements are the main micro-areas, where external and internal stresses are redistributed. Transformation of contact types, their amount and area in the course of deformation, appears to be the principal factor controlling the clay soil behaviour.

The analysis of the entire complex of factors permits the conclusion that understanding the regularities of clay behaviour under different loads and different physico-chemical conditions requires deep understanding of the inner structure of soil and the processes of its response to these impacts. Therefore, further success in soil and rock mechanics in many respects will be determined by the progress in understanding the nature of processes operating inside soils. This requires a transition from phenomenological to the essential study of soils. Physico-chemical soil mechanics is the main basis for such a transition.

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FIGURE CAPTIONS

Fig. 1. Contracting effect of the water meniscus between two spherical particles.

Fig. 2. Capillary water rise in capillary pores (a) and the water meniscus in a capillary tubular pore (b): 1 - pore walls; 2 - capillary water.

Fig. 3. Formation of structural repulsing forces between flat surfaces upon the overlapping of bound water films: 1 - particle; 2 - osmotic water; 3 - bound water film; 4 - zone of overlapping between adsorptional water films.

Fig. 4. Isotherm of wedging pressure in the boundary hydrate film (after Deryagin & Churaev, 1984).

Fig. 5. Bubble method of obtaining the isotherm of wedging pressure in a hydrate film (after Deryagin & Churaev, 1984): 1 - a solid body, 2 - bound water, 3 - free water, 4 - air bubble.

Fig. 6. Wedging pressure isotherm of water β-film at the surfaces of water, quartz, and mica: 1 – water on mica; 2 – water on melt quartz; 3 – $2 \cdot 10\text{-}5$ N KCl solution on quartz; 4 – $1 \cdot 10\text{-}4$ N KCl solution on glass; 5 – water on quartz. (after Deryagin, 1986)

Fig. 7. Energy types of contacts in dispersed systems: (a) coagulational, (b) transitional (point), (c) phase; 1 – a mineral particle, 2 – bound water.

Fig. 8. Schemes of globular (a) and bidisperse (b) models of disperse porous structure: N – average number of particles from node to node depending on the structure porosity; r – average radius of particles (globular model) and radius of fine particles (bidisperse model), R – average radius of coarse particles (bidisperse model) (after Rebinder *et al.* 1964 and Sokolov, 1991).

Fig. 9. A scheme of the 'twisting playing card house': 1 – a general view; 2, 3, 4 – different stages of structure transformation on compaction, where a – is the particle length, b – is the particle thickness and θ is the angle of particle inclination (after Sokolov, 1991).

Fig.10. Area of coagulational (a), transitional (b) and phase (c) contacts: 1, 2 – particles, 3 – a hydrate film of bound water.

Fig. 11. Clays soils with open (a) and closed (b) porous systems: 1- mineral particles, 2 – bound water, 3 – free water.